

A New Universal Generalized Method for Multifractals Synthesis

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Ivan G Grabar* and Olena S Kilnitska

Polissia National University Staryi Blvd 7, Zhytomyr, 10008 Ukraine

***Corresponding Author:** Ivan G Grabar, Polissia National University Staryi Blvd 7, Zhytomyr, 10008 Ukraine.

Abstract

For the first time in the world, a universal generalized method for mono- and multifractals synthesis is proposed, which allows combining and reproducing almost all known artificial and semi-artificial methods of fractal synthesis on a single natural platform - Brownian point dynamics (BPD) in the field of N gravitational forces.

On the basis of the BPD method, a set of algorithms for the synthesis of fractal sets of a wide range of characteristics and properties, such as monofractal, multifractal, isotropic, anisotropic, rectangular, hexagonal, ring, single- and multi-loop, etc., has been developed.

The fractals synthesized by the BPD method enable the production of solid samples of these fractals with predetermined characteristics using modern 3D printing technologies and experimentally study their porosity, fractal dimensions, residual strength, aerodynamic and hydrodynamic drag, etc.

It is shown that the Brownian dynamics of a point in a field of N gravitational forces clearly demonstrates the generation of a fractal order by Brownian chaos with a cascade of discrete levels ordered by scaling, i.e., an important philosophical result of the demonstration of order from chaos. The influence of minimal values of N on the synthesis of linear, planar or volumetric multifractal sets is demonstrated.

Keywords: chaos; multifractal; synthesis of multifractals; tensor structure of multifractals; inverse problem of fractal analysis

Introduction

The method of Brownian point dynamics (BPD) in the field of N gravitational forces, as a new universal natural method of synthesis of fractal structures with a predetermined set of utility characteristics, in the opinion of the authors, has wide perspectives in the future. This is a new scientific field that is rapidly developing and extending into the field of materials science (creation of new ultralight materials with an infinitely large free surface, good noise and heat insulation properties), radio engineering (creation of multifractal antennas), signal transmission and processing theory (fractal image of their attractors allows compressing the signal image thousands of times), etc. The authors propose a set of algorithms and 3D technologies for the synthesis of multifractal structures with predefined properties.

The developed method of synthesis - chaos splitting in the field of N attractive forces - makes it possible to delve into the formation of the kinetics of a large number of generations (structural levels) of multifractals. The BPD method in the field of N forces suggested unprecedented new synthesis possibilities, such as isotropic and anisotropic fields, variation of the coordinates of the points of application of the attractive forces, perturbation of these coordinates, control of the transparency of force barriers, etc. This, in turn, made it possible for multifractal structures with a small number of phases to obtain the contrast of these phases against the background of others. And this, in turn, gave the key to the construction of the tensor-matrix formalism of multifractals. The tensor-matrix formalism made it possible to obtain quantitative ratios and engineering formulas for calculating the same utility characteristics. This result made it possible to formulate and solve inverse problems of multifractal synthesis, i.e., the synthesis of multifractal structures with predetermined properties.

Analysis of publications

Many new scientific publications have been devoted to the practical application and modeling of physical processes using fractal approaches, among which these publications are of importance [1-14]. Thus, [1] demonstrates a significant breakthrough in the design of new generation fractal antennas. In [2], the authors have demonstrated the effectiveness of using a new set of synthesis, clustering, and packaging algorithms based on fractal representations, which significantly increases the efficiency of using complex structures. In [3], fractal geometry approaches are used to create digital images of natural surfaces with the aim to model electromagnetic scattering. In [4], a physical approach is proposed for the synthesis of fractal surfaces in remote sensing applications. In [5], the relationship between fractal arrays and their ability to generate radiation patterns is investigated and it is shown how fractal models allow control the parameters of radiation patterns.

In [11-12], the effectiveness of creating a special mathematical apparatus for modeling the dynamics of liquids and gases in a fractal formulation is shown. Among the most active groups in the creation of a modern mathematical apparatus for fractal analysis is a group of scientists at the National Polytechnic Institute, Mexico City. This group is headed by a well-known scientist, Professor Alexander S. Balankin (Profesor, Instituto Politécnico Nacional, México) [11-12].

In [13], crucial scientific results were obtained that demonstrate the relationship between random processes and fractal geometry in the most general case.

The paper [14] is a classic work that set the importance of fractal representations for the next few decades.

However, the works [1-5; 11-14] lack universal general approaches that would demonstrate the naturalness of the processes of synthesis of mono- and multifractals, which, in our opinion, hinders the physical perception of fractal geometry as a general physical phenomenon with an absolutely natural origin. Such an understanding of fractals would help to create universal algorithms for the synthesis of multifractals with incredibly large capabilities, including the correct formulation and correct solution of the inverse problem of fractal analysis.

In [6-10], for the first time, an attempt was made to show the universal general nature of multifractal synthesis as a natural process of BPD in a field of N gravitational forces, and to show how in these models the chaotic initial conditions create the conditions for the synthesis of multifractal sets, thus to demonstrate the splitting of chaos into a fractal superorder. These are the aims of our article.

Main results

The proposed method of BPD synthesis of multifractal structures allows to synthesize almost all known fractal structures. Specific examples will be given below.

The statistical drawing by the BPD method in the field of N gravitational forces creates great favorable conditions for purposeful synthesis of multifractals, since it allows the use of a large number of control parameters, which is very convenient in engineering problems. For example, by varying the number of attraction forces N and controlling the anisotropy index η , we can obtain:

- For N=2 - a mono-($\eta=1$) or multifractal ($\eta \neq 1$) Cantor (Fig. 1);
- For N=3, a mono-($\eta=1$) or multifractal ($\eta \neq 1$) Sierpinski triangle (Fig. 2);
- For N=4 - flat or bulk mono-($\eta=1$) and multifractal ($\eta \neq 1$) structures, for example, MAX-4 (Fig. 3);
- For N=9 - mono-($\eta=1$) or multifractal ($\eta \neq 1$) fractals - Serpinsky carpet, Wisek, Koch fractals (Fig. 4)

Increasing N allows us to obtain almost all known mono- and multifractal structures in one-, two-, and three-dimensional fractal structures.

In addition to controlling the parameters, it is possible to control the coordinates of the points of application of the gravitational forces:

$$\begin{aligned}
 &A_1[X_1 Y_1 Z_1]U[m_{1x} m_{1y} m_{1z}] \\
 &A_2[X_2 Y_2 Z_2]U[m_{2x} m_{2y} m_{2z}] \\
 &\dots\dots\dots \\
 &A_N[X_N Y_N Z_N]U[m_{Nx} m_{Ny} m_{Nz}]
 \end{aligned}$$

In addition, it is possible to control the transparency of the barriers ϵ of the attraction forces, perturbations of their values, perturbations of the coordinates of the points of attraction, etc.

Then, for N=10 and 10 levels of values of each of the coordinates, as well as only 2 levels of field parameters η and ϵ , we have the total number of possible synthesized structures:

$$Q = 5 \cdot 10^{30} \cdot 2^{30} \approx 10^{40}$$

which roughly corresponds to the number of atoms on planet Earth.

The BPD synthesis method allows us to implement the most daring hypotheses about fractal structures of Gregor Cantor, Wencelas Serpinski, Henri Poincaré, Henry Hacken, Ilya Prigozhin and other scientists who have researched, studied, systematized and constructed fractal sets. The creation of the BPD synthesis method completes the stage of heuristic fractal design, and begins the stage of conscious synthesis of fractal structures with predefined properties. A fractal becomes a familiar object with a reasonable physical nature.

From our point of view, a fractal is a material object generated by the Brownian dynamics of a material point in a field of N gravitational forces when they are discretely perturbed randomly, and this is a physical reality! In this case, the forces can be: central, potential, elastic, attraction or repulsion. There is only one limitation - the generation of their inclusion must be random, and only one force acts at each discrete moment.

The practical implications of the development of fractal representations promise new solutions to a number of applied problems: creation of fractal models of optimal production of liquid and gaseous hydrocarbons, creation of biocompatible materials and their controlled fusion with biotissues, creation of percolation-fractal surfaces for implants, filtering, separation and purification of liquids and gases, spraying of liquids in a number of technologies, control of turbulence of liquids and gases in power plants, aviation, compressor and space technology, effective fire extinguishing, creation of effective models of the geology of gold nuggets, amber and other deposits, creation of new antenna systems in radio engineering and electrodynamics, etc.

All these are the tasks of today, which are solved by the methods of fractal analysis within the framework of percolation-fractal materials science [6].

Our algorithms allow the synthesized multifractals to be converted into 3-D printing technologies. In turn, the availability of a large number of solid-state fractal structures will significantly increase experimental research and stimulate further development of both fractal technologies and the theory of fractal analysis. We have mastered the synthesis of various fractal structures: mono- and multifractal; isotropic and anisotropic; orthogonal and oblique or chaotic; interpenetrating multifractal subsystems; with control of the transparency of central force barriers, etc.

Examples of multifractal synthesis

The results of the BPD-synthesis of mono- and multifractals are shown in Figs. 1 and 9.

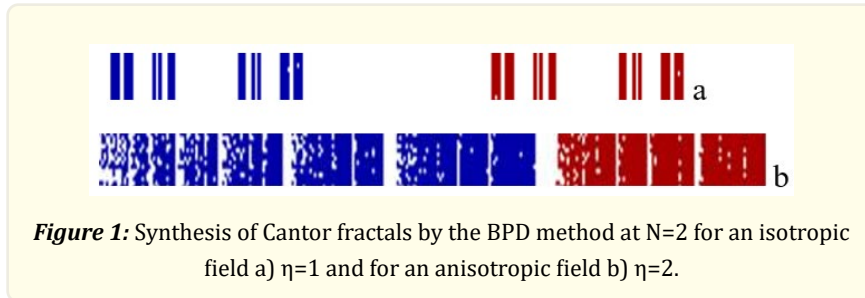


Figure 1: Synthesis of Cantor fractals by the BPD method at $N=2$ for an isotropic field a) $\eta=1$ and for an anisotropic field b) $\eta=2$.

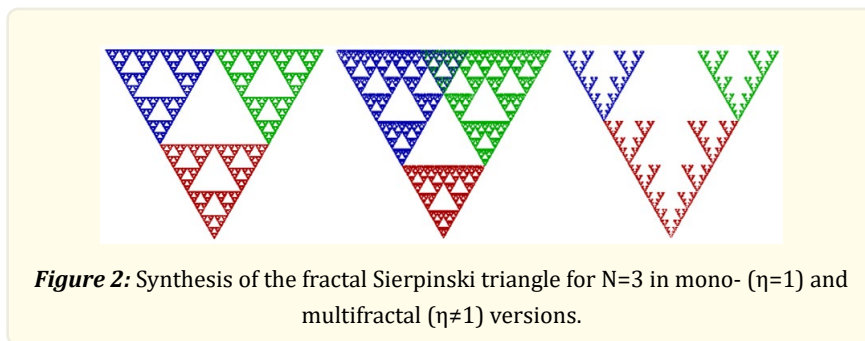


Figure 2: Synthesis of the fractal Sierpinski triangle for $N=3$ in mono- ($\eta=1$) and multifractal ($\eta \neq 1$) versions.

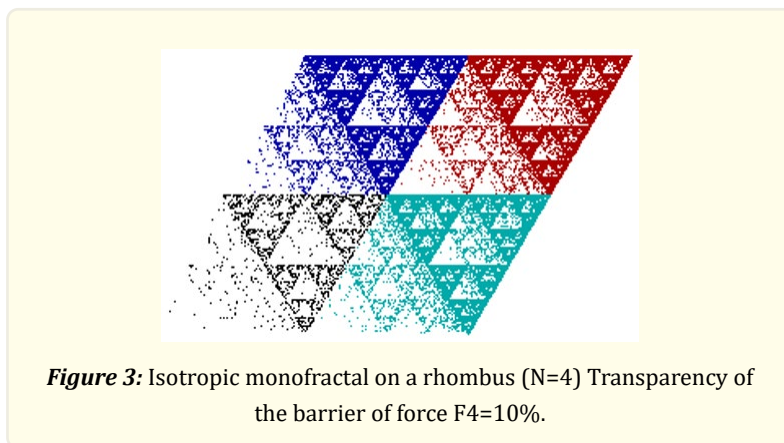


Figure 3: Isotropic monofractal on a rhombus ($N=4$) Transparency of the barrier of force $F_4=10\%$.

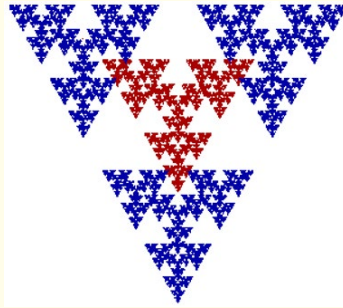


Figure 4: Prof. Grabar's monofractal triangle (N=4).

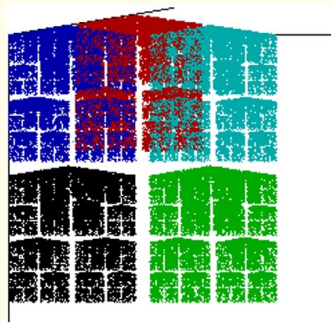


Figure 5: An example of BPD synthesis of a bulk fractal by the BPD method (N=5).

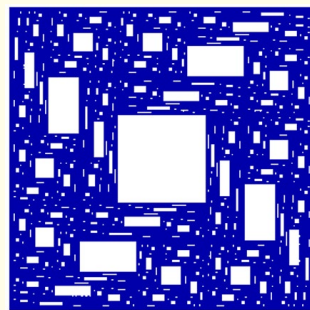


Figure 6: An example of BPD synthesis of a multifractal anisotropic skewed lattice (N=4) MAX-4.

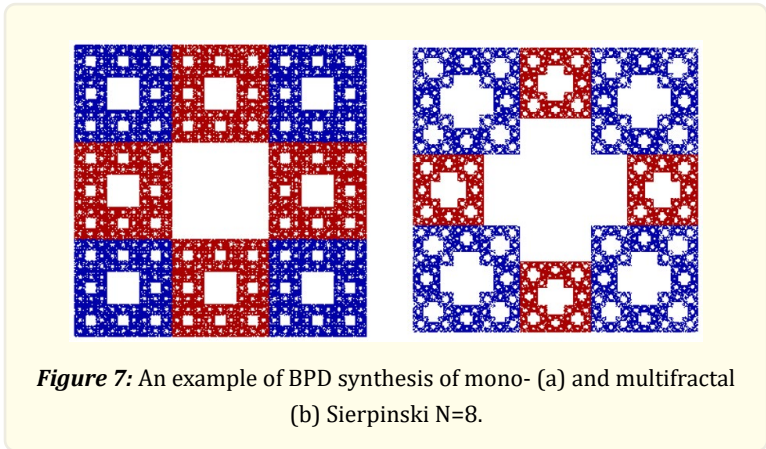


Figure 7: An example of BPD synthesis of mono- (a) and multifractal (b) Sierpinski N=8.

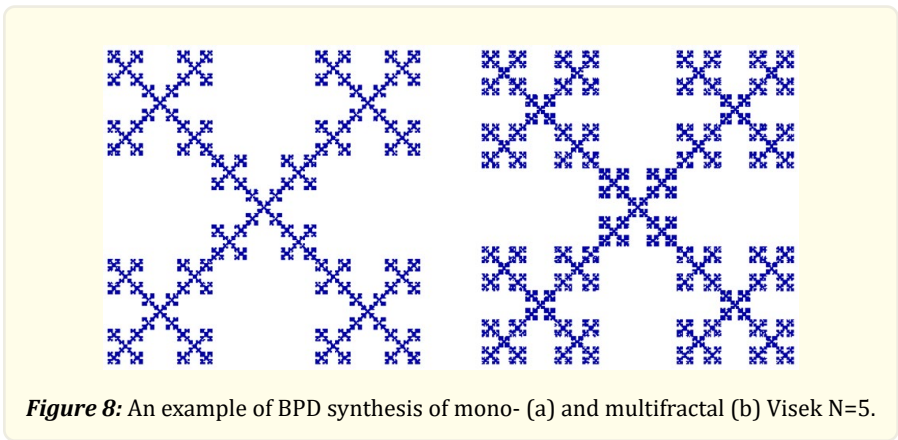


Figure 8: An example of BPD synthesis of mono- (a) and multifractal (b) Visok N=5.

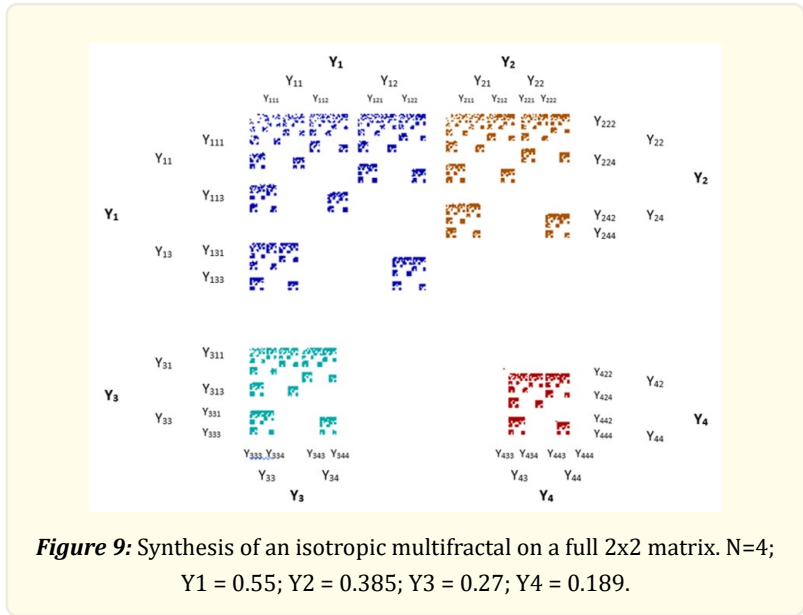


Figure 9: Synthesis of an isotropic multifractal on a full 2x2 matrix. N=4;
Y1 = 0.55; Y2 = 0.385; Y3 = 0.27; Y4 = 0.189.

The basics of the tensor-matrix formalism of multifractals are given in [6-7]. Using them, let us determine the sizes of the subsets of the 2nd, 3rd and subsequent generations of the multifractal (Fig. 9), given the sizes of the subsets of the first generation: $Y_1 = 0.55$; $Y_2 = 0.385$; $Y_3 = 0.27$; $Y_4 = 0.189$

Then for the 2nd generation we have:

$$Y_{11} = Y_1 * Y_1 = 0.55 * 0.55 = 0,3025$$

$$Y_{12} = Y_1 * Y_2 = 0.55 * 0.385 = 0,212$$

$$Y_{13} = Y_1 * Y_3 = 0.55 * 0.27 = 0,149$$

$$Y_{14} = Y_1 * Y_4 = 0.55 * 0.189 = 0,104$$

.....

$$Y_{44} = Y_4 * Y_4 = 0.189 * 0.189 = 0,036$$

Accordingly, the characteristic sizes of the subsets of the 3rd generation of the multifractal are calculated as follows:

$$Y_{111} = Y_1 * Y_1 * Y_1 = 0.55 * 0.55 * 0,55 = 0,166$$

$$Y_{112} = Y_1 * Y_1 * Y_2 = 0.55 * 0.55 * 0,385 = 0,116$$

$$Y_{122} = Y_1 * Y_2 * Y_2 = 0.55 * 0.385 * 0,385 = 0,0815$$

.....

$$Y_{444} = Y_4 * Y_4 * Y_4 = 0.189 * 0.189 * 0,189 = 0,0068$$

Similar calculations can be easily performed for the 4th, 5th and subsequent generations of the multifractal (Fig. 9). These calculations with an error of no more than 1% coincide with the experimental data (Fig. 9).

Fig. 10 shows a fragment of 3D printing of the BPD -synthesized multifractal anisotropic skewed grating (N=4) MAX-4 (Fig. 6). Solid-state samples of BPD -synthesized multifractals allow us to study the aerodynamic, hydrodynamic, elastic characteristics of multifractals, their residual strength, porosity, heat and sound insulation properties, free surface parameters, etc.

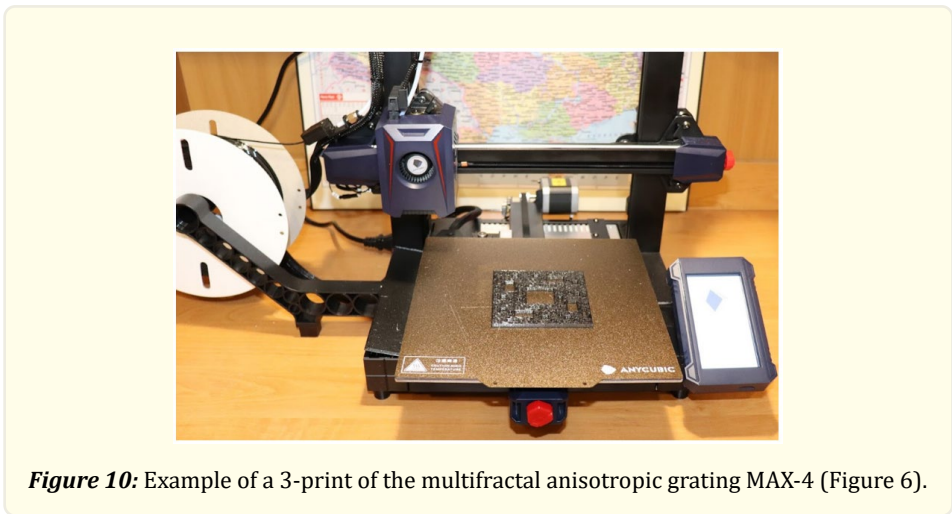


Figure 10: Example of a 3-print of the multifractal anisotropic grating MAX-4 (Figure 6).

Summary

In the more general case of synthesis, our algorithms allow us to use the tensors of the field of N gravitational forces as control parameters for the synthesis of multifractals [6-7]:

- Tensors of the gravity field parameters

$$M_i^k = \|\| m_1^x m_1^y m_1^z m_2^x m_2^y m_2^z m_N^x m_N^y m_N^z \|\|$$

- Transparency tensors of gravity barriers

$$\varepsilon_i^k = \|\| \varepsilon_1^x \varepsilon_1^y \varepsilon_1^z \varepsilon_2^x \varepsilon_2^y \varepsilon_2^z \varepsilon_N^x \varepsilon_N^y \varepsilon_N^z \|\|$$

- Frequency tensors of coordinate perturbations $A_N(X_N Y_N Z_N)$

$$\Omega_i^k = \|\| \omega_1^x \omega_1^y \omega_1^z \omega_2^x \omega_2^y \omega_2^z \omega_N^x \omega_N^y \omega_N^z \|\|$$

- Tensors of coordinate perturbation amplitudes $A_N(X_N Y_N Z_N)$

$$R_i^k = \|\| r_1^x r_1^y r_1^z r_2^x r_2^y r_2^z r_N^x r_N^y r_N^z \|\|$$

Conclusions

1. A universal generalized method for the synthesis of mono- and multifractal sets is proposed for the first time. Unlike existing synthesis methods, the BPD method is a natural process of 'splitting' chaos in a field of N gravitational forces.
2. In addition to the above-mentioned universality and generality, the BPD method allows to control the synthesis of fractal sets deliberately and purposefully, with a predetermined set of service-useful properties of fractals. This is what gives us the right to assert the possibility of solving the inverse problem of fractal analysis.
3. The main groups of factors influencing the results of the synthesis of mono- or multifractals are identified and systematized. The proposed algorithms for synthesis control are brought to engineering technologies.
4. It is proved that all generations of synthesized multifractals are quantitatively described by tensor relations. The tensor-matrix formalism of these relations is constructed and engineering formulas for calculating the sizes of subsets, areas, volumes and higher degrees of subsets of j-th generations of synthesized multifractals are obtained. This greatly simplifies the synthesis of multifractals with a predetermined set of service-useful characteristics.
5. It is shown that it is the Brownian order of inclusion of the attractive forces that generates a multifractal, while periodic sequential inclusions of the attractive forces generate limit cycles.
6. We believe that the design, synthesis and experimental study and application of multifractals in modern technologies is entering a new stage - the purposeful synthesis of multifractals with predetermined properties.

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